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IN TRANSITION: MATHEMATICS AND ART

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in memory of Vaughan Jones

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Abstract. Aalto University has been able to create a fruitful frame for activities enhancing interaction between mathematics, art and architecture. One of the most recent highlights of this progress was a student exhibition titled 'IN TRANSITION – Mathematics and Art' [**12**] at Espoo Cultural Centre [**9**]. The exhibition was further extended to the Aalto Math&Arts exhibition in Shanghai ([**3**],[**25**]). In this paper we describe our long-lasting open-minded collaboration to build a minor in Mathematics and Arts that is useful from freshmen to PhD students across the conventional barriers between disciplines. A dialogue between scientific and artistic practices break clichés related to mathematics by bringing deep phenomena in the field to the level of human experience. Challenges and future scenarios are discussed broadly.
Some ideas about the present state of Aalto Math and Arts can be found in [**2**].

Figure 1. Aalto Math&Arts logo designed by Taneli Luotoniemi.

[0]This article including high resolution pictures can be downloaded from shorturl.at/klnrD
2010 *Mathematics Subject Classification* 00, 51, 53, 97.
Key words and phrases: mathematics; art; architecture; low-dimensional geometry and topology; exhibition.

FIGURE 2. Exhibition poster designed by Taneli Luotoniemi.

1. The background of Aalto Math and Arts

To enhance fruitful interaction and new systematic ways of acting between contemporary mathematics and arts in different levels of education and research were established at Aalto University since its foundation at 2010. Article [**22**] reveals in detail some early steps towards multidisciplinary activities challenging the traditions and communication in the field. Starting from the very first ambitious 10cr course implementation of Crystal Flowers in Halls of Mirrors: Mathematics meet Art and Architecture [**7**] we have gained a tremendous amount of experimental knowledge about possibilities, challenges and ways to influence along the way. Multitalented students from diverse backgrounds that have found our courses, as well as time in their schedule to participate and bring their own interest and expertise, have had a key role in making the progress possible and enrich the contents.

During the academic years 2018-2019 we piloted a bachelor and master level 25cr Math&Arts minor program at Aalto University. The contents of the courses are chosen based on our experience on earlier Crystal Flowers. There are no prerequisites in mathematics or arts in this program. We focus on topics that are visually approachable to students without calculus tradition but interesting and new also to PhD students in mathematics or engineering. The minor is designed for students from all schools of Aalto in all levels of education. This systematic approach to engage mathematics and arts is open to students from the other universities in Finland as well. Especially art educators and students from teacher education are highly encouraged to take these courses. School teachers are also welcome to update their skills and find new concrete multidisciplinary ideas for their daily work. Courses can also be taken independently, but either (or both) of the 5cr courses below are recommended to be finished before the 10-15cr Crystal Flowers course MS-E1000. There are no exams in this minor, but the grading and working is based

on active participation on contact teaching, reflections, exercises, essays, demonstrations, discussions, workshops and other tasks. The focus is on the treatment of the subject, artistic inventiveness, and the articulated and careful execution.

Many teachers and visitors from different disciplines make together the courses. D.A. Taneli Luotoniemi from Art Education has been involved from the very beginning. His beautiful thesis [**21**] contains many ideas that were tested during the course. MaA Laura Isoniemi from textile and surface design has brought wonderful insight and practical experiments to the courses. Her recent book [**15**] reveals some aspects of her methods that were used during the courses. Professor Toni Kotnik from architecture and engineering and Professor Tapio Takala from computer science have enriched the contents in many ways over the years.

The 5cr course MS-C1001 Shapes in Action is to be lectured every other year. There the students will learn to recognize visual patterns, structures and shapes of our natural and cultural environment, and analyze their symmetries as sets of actions, like rotations, reflections, and transformations. Mathematically rigorous treatment is used to luminate the elegant structures and shapes to an intuitive perspective, and afford the students with a widened scope of possibilities especially within practices involving modular repetition. By the end of the course, the students will be able to distinguish aspects from their own fields which can be presented, investigated, and developed using the language of modern mathematics. The first implementation of this course was taken by students broadly from different disciplines, but the novel Data Science program students become the major group. This was mainly because the course was advertised as part of the program and also the contact teaching was fit to their schedule. The content of the course is topics on planar, spherical and hyperbolic 2D symmetries, Kleinian groups, conformal dynamics, 3D geometries, manifolds, orbifolds and fractals. This course provides views on research level differential geometry to broad audience. During the course we will consider methods offered by various fields of mathematics which meet needs in art and architecture. Through concrete projects, we will find phenomena and interpretations of these phenomena from both classical and modern mathematics. We hope to describe the content, approaches and the outcomes of the first implementation of this course in another article elsewhere.

The 5cr bachelor level course UWAS-C0014 Spatial Structures is lectured on annual basis. During the course students will learn to understand some of the spatial constraints governing visual making, how they relate to each other, and what are the geometric and topological concepts used to refer to them. The students will learn how to represent structures and spaces in interesting and effective means. They will also learn to recognize and contextualize various geometric and topological phenomena, talk about them using appropriate vocabulary, solve spatial problems arising from their own practice, and know where to find further information about them. Dealing with the properties of polygons, meshes, solids and their projections, many contents of the course are perfect for application in digital technologies such as programming, CAD and 3D printing. Topics discussed include tilings and patterns, polyhedra, space-fillings, symmetries, projections, perspective, topology, hyperbolic and projective geometries, knots, 4-dimensional space, geometry in fiction, and mathematical art.

The 10-15cr course MS-E1000 Crystal Flowers in Halls of Mirrors: Mathematics meets Art and Architecture is lectured every other year for students from bachelor to doctoral level. During the course we will consider methods offered by various fields of mathematics which meet needs in arts and architecture. Through concrete projects, we will find phenomena and interpretations of these phenomena from both classical and modern mathematics. Various topics especially from low dimensional topology and geometry. The major outcome of this course is an exhibition. In 2019 implementation it was created in collaboration with Espoo Museum of Modern Art EMMA [**8**] in Espoo Cultural Centre [**9**]. This process is explained in more detail in the following chapters.

The learning outcomes of the whole Math&Arts minor are summarised in what follows. Students will learn to find connections between mathematics and art and architecture. Real mathematics will be revealed through patterns, symmetries, structures, shapes and beauty in such a way that will enable the student to view our environment from a new perspective. By the end of the course, the students will be able to distinguish aspects from their own fields which can be presented, considered and developed using the language of modern mathematics.

Based on the pilot, the Aalto Math&Arts Minor was made an official Aalto minor for academic period 2020-2021. Thanks on the most recent administrational progress at Aalto University go to Dean of Science School Jouko Lampinen, Vice Dean of School of Arts, Design and Architecture Saija Hollmén, Professor Martti Raevaara from Aalto Arts Education and Professor Nuutti Hyvönen, the department head of Aalto Mathematics and Systems Analysis.

2. Crystal Flowers 2019 in Espoo

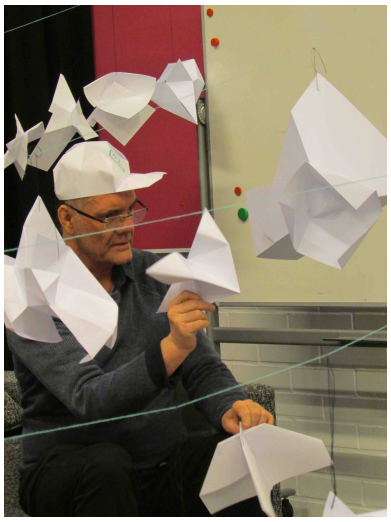
The plans for the exhibition were started already in early fall 2017 when Chief Curator Reetta Kalajo from EMMA [**8**] got interested in collaboration. EMMA museum is conveniently located close to Aalto University and there has been many joint projects and events where students, researchers and alumni of Aalto have contributed. As independent student exhibitions without a leading professional artist were out of the question, Kalajo suggested Espoo Cultural Centre [**9**] as a potential venue. Nearby locating, beautiful building designed by architect Arto Sipinen (1936-2017) and build 1989, that would celebrate its 30th anniversary the same year as our exhibition would take place, turned out to be a wonderful possibility for us. The architecture of the building is full of surprises and interesting details to explore. Espoo Cultural Centre is the main scene for performing arts and cultural life in Espoo. The Espoo city orchestra, the Espoo Theatre, April Jazz Festival, Espoo Ciné Festival, PianoEspoo and ChoralEspoo Festivals are on stage on a regular basis. Public library, cafeteria, good premises for students from nearby schools, various society members, hobbyists and random visitors make it an ideal meeting point for diverse groups of people from early morning to late evening. The idea that our math and art exhibition would take place in a venue, where people would come free of charge, accidentally, just on the way to do something else, was very inspiring. The Cultural Producer Sanna Katajavuori at Cultural Centre immediately bought our idea and the decision to nail our exhibition venue was done.

For curating the exhibition and tutoring the students we hired consultant and designer MaA Pablo Riquelme and student Markus Holste from Aalto Interior Architecture. Riquelme's motto 'I believe design can help us to generate empathy and positive interaction in urban settings and social structures.' fitted nicely to our goals to try to reveal mathematics in positive interaction with arts. Marco Rodriguez, the curator and exhibition designer of our earlier exhibition in Heureka Science Centre in 2017 was also available for us during the process. The Cultural Centre premises at hand were soon recognized more challenging due to greater freedom in shape and other possibilities. The inspiring premises and facilities of the Aalto Design Factory (DF) [**1**] were again provided to us by Kalevi Ekman and his helpful staff. Together with Luotoniemi, Isoniemi, Kotnik and Takala we started the lectures in January 2019 in a similar spirit as earlier implementations, but this time we were more focused on topics that would give good seeds and preparatory material to the final exhibition. Production Manager Sami Supply and Chief Curator Arja Miller from EMMA also joined our teacher team. It was a fortunate coincidence that artist Alicja Kwade had her exhibition [**20**] at EMMA during the same time, and our students had a possibility to learn, not only the exhibition itself, but they also got a glimpse behind the scenes. How is it to build an exhibition and how to collaborate with an artist from the perspective of a museum. Kwade's ways of working with details and showcasing the process of the stand-alone art pieces is very similar to the process in Math and Arts courses that we have had in mind independently. PoP Hannu Hyyppä at Aalto ENG Measuring and Modelling for the Built Environment who is running a laser scanning CoE of Finnish Academy provided his support in producing a virtual exhibition on our exhibition. PhD student Juho-Pekka Virtanen with his teammates gave a presentation about the possibilities of modern technology and produced a virtual exhibition that is available in [**14**]. The exhibition poster in figure 2 is designed by making use of the point cloud of laser scanned lobby at Cultural Centre. Artist Lauri Astala [**4**] visited the course and gave an influential presentation about his beautiful artworks and thoughts about mathematics and arts. During the course, intensive three-hour meetings were held with the students twice per week. We divided the students into diverse groups of six students at the very beginning to allow them to slowly get to know one another. More details on the pedagogy and approaches of the lectures can be found in the article [**23**].

3. Workshops

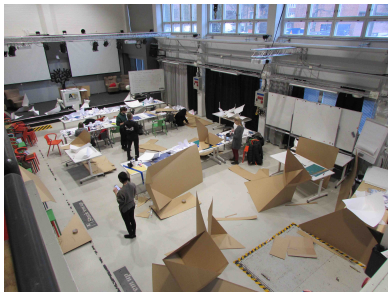
Two workshops were organised during the course. Origami artist Paul Jackson [**16**] joined us in February. The three full day workshop was a beautiful process artistically, mathematically and educationally. During the first day Jackson explained students a simple idea on how to invert a corner so that a similar corner fits in. Standard A4 paper was used for this part. Students were encouraged to experiment with different types of corners, while rest of the parts of the pieces were left without restrictions. By the end of the day, students collected their outcomes to be proceeded into next level the following day. Jackson gave critics and encouragement for students' proposals.' The next day students moved into milk board. It was very interesting to see how stronger material gave new directions to the shapes. Students started to work in small groups, trying to find best candidates to

be enhanced into the final scale. During the third day students were working with the final material, B-flute card board, that is 5mm thick. This material and scale turned out to be very challenging. As card board is not that homogeneous as milk board, students needed support in finding good solutions. Jackson's way of working with the students was very good, since he left a lot of space for participants own ideas.

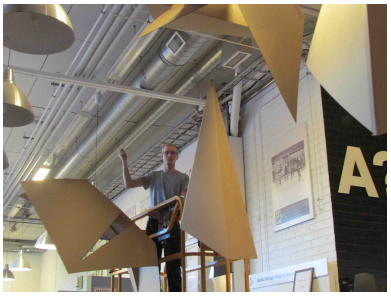


(A) Some copy paper proposals.

(B) A milk board proposal.



(C) Working with card board.



(D) Pauli Hanhilahti support at DF.

(E) Painting in action

(F) Pieces ready for assembly.

At the same time, the given restrictions were guiding towards reasonable solutions. By the end of the day, all pieces were hanged to the ceiling of Design Factory. The plan was to paint the pieces afterwards, before the final hanging at Espoo Cultural Centre. This activity took place at Design Factory under guidance of Riquelme and Holste as instructed by Jackson. The outcomes of the workshop are described poetically by Jackson: Despite best intentions, relationships — whether professional or personal — are rarely equal. They succeed when everyone is given their own space, each mesh- ing closely against the other, in endless variations of give and take. Paul Jackson has been a professional paper folder and origami artist since 1983. He divides his time between teaching 'Folding' to students from many design specialisms, writing books (his 2011 book 'Folding Techniques for Designers' [**19**] is a best seller), creating folded artworks and being a consultant for a great diversity of projects. He has an MA in Fine Art from University College London.

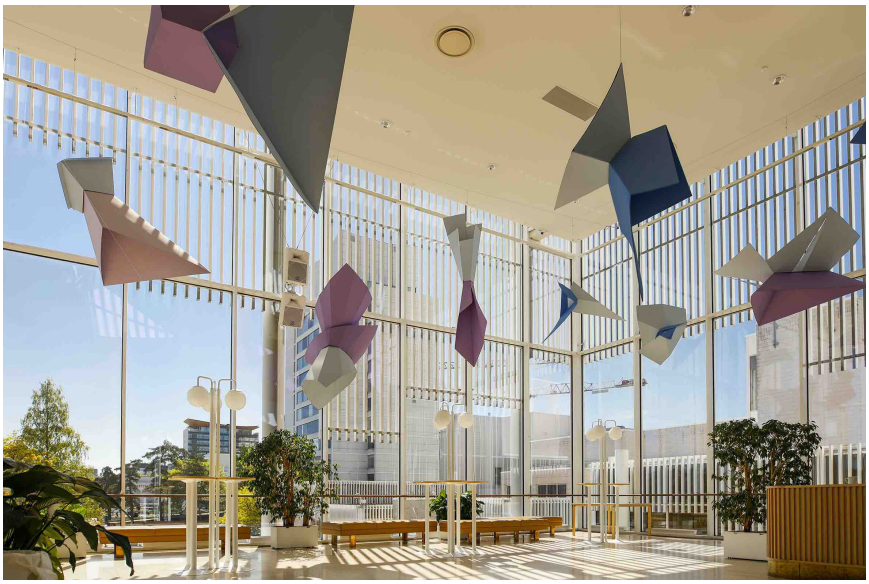


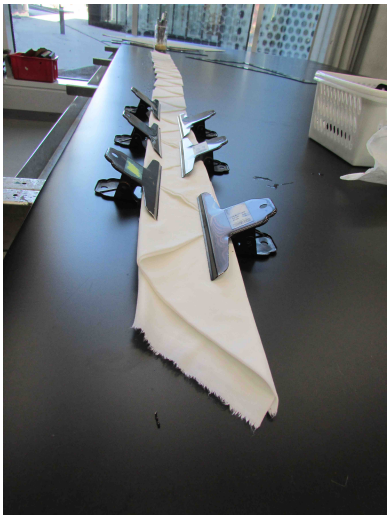
FIGURE 4. You and I. Photo: Kalle Kataila

Laura Isoniemi organised a textile workshop that had two parts. The first one was focusing on Textile design, dyeing and mathematics. Surface patterns, whether they are in textile, on paper or built environment, follow a rhythm. The rhythm is based on symmetry, reflections and repeats of various kinds. In textile tie and dye- method the figure is born with folds and preventions. The folds are repeated and then the textile is blocked or tied with a tool to prevent the colors to absorb in certain places. Works were done with one color, in order to maximize the effect of the rhythm.

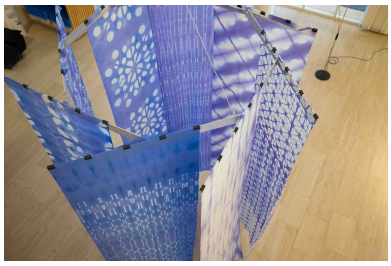
The other part of the surface design of textiles was titled Digital textile printing added with mathematics. Digital printing offers a lot of possibilities for textile design in terms of benefitting from the use of photos and their different qualities. The digital printer works as a copy machine, but prints the outcome on textile. The

technique is the most ecological as it is using only the needed amount of colour. The main advantage is, that digital printing offers the possibility to use various artistic techniques: big scaling and endless photo manipulations with a computer. In the printed shirts, the surfaces are created for example with a picture, where there is a dog in a ribbon or a building site. With the help of symmetry laws, they are turned into fascinating surfaces. The task was to create a surface out of student's own picture. Each fabric has its own fingerprint and verification how symmetry, colours, scaling and creative mind can turn any subject into a beautiful surface. Most of all, the format of a shirt shows, how the patterns work in a 3D on body. Pictures from the textile workshop are provided in figure 5.

(A) Some tools for the textiles workshop.



(B) A folded piece ready for a color path.



(C) Fold, Tie and Dye Textiles. Photo: Kalle Kataila

(D) An outcome by Yi-Chiao Tien from Digital Printing Workshop.

FIGURE 5. Laura Isoniemi workshop.

4. Mathematics and Arts Colloquium

The idea of organising public Math&Arts Colloquium talks in connection of Math&Arts courses serves several purposes. On one hand, international visitors

that bring interesting contemporary topics to broad audience are highly appreciated not only among school children, school teachers and other target groups of different outreach programs run at universities, but also random curious minded people interested in true interaction of science and arts. All these talks are videotaped and collected to our webpage [**2**]. They have turned out to be valuable educational material in different contexts. Visitors are often interested to give more focused lectures or focused workshops to our students. This interaction gives also valuable insight to researchers at the university about possibilities to see what is happening in other disciplines.

Paul Jackson's colloquium talk can be found online in Aalto Youtube [**18**]. In addition to his colloquium talk, Jackson provided a 'how to' type short video about his genius idea to build a cube from origami modules and how to invert the corners to produce new polyhedrons [**17**]

Thomas Banchoff was the other speaker. His talk can be found online in Aalto Youtube [**5**]. He acted also as an opponent to Luotoniemi's defence as an expert on 4D topology.

Mathematics and Arts Colloquium in connection with a multidisciplinary course like this is a well-functioning concept. Public talks related to ongoing new activities at the university provide an accessible and convenient channel for a broad audience. Art gives a fruitful frame to make mathematics more visible in the society. The Colloquium was supported by Niilo Helander Foundation.

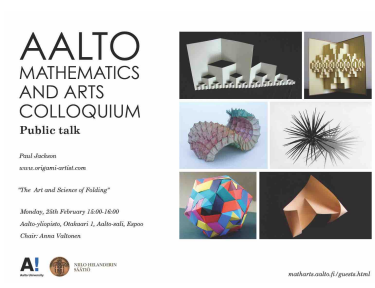
5. Exhibition tasks

Our earlier exhibition at Heureka Science Centre in 2017 and its preparatory tasks are explained in detail in article [**23**]. That experience provided us a really good starting point and reference for the next exhibition, but of course, the circumstances in the Cultural Centre were in many ways completely different. Anyways, the timetables and all material were indeed useful for this new task. The biggest change we made, was the conscious decrease in the teaching material, to make more space for the exhibition planning. Not all students had taken our smaller courses, but the Spatial Structures course was lectured in parallel with Crystal Flowers, and some student took that also at the same time.

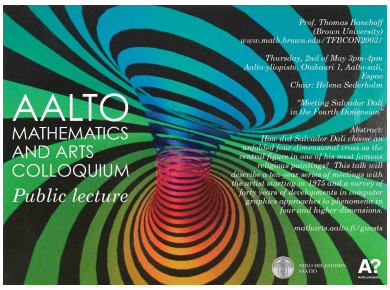
As in the Sensual Mathematics case, the exhibition content was created by groups of multidisciplinary students. Groups were formed by the teachers using criteria based on diversity. Each group made interpretations on low-dimensional geometry and topology by designing a sculpture for the exhibition. The exhibition space included a not only about a 150 square meter gallery space in two floors, but also the numerous possible spots in spacious lobby that occupied several floors. A good idea about the exhibition site is found in [**9**].

Students where highly encouraged to take the architecture of the exhibition site into account in their design. This turned out to be challenging in many ways. As the building has varying, complex shape, it did not actually restrict ideas but rather gave more possibilities for them.

A priori, the students were allowed to use any media, materials and experiment with them. As in all exhibitions, there were some guidelines to follow. More details were discussed in the tutoring sessions that were organised on a regular basis with each group. Each team had a 1000 euro upper limit for the budget, including the



(A) Math&Arts Colloquium by Jackson.



(B) Math&Arts Colloquium by Banchoff.

(C) Thomas Banchoff lecturing on Salvador Dali. Photo: Mikko Raskinen

FIGURE 6. Math&Arts Colloquium.

material for prototypes and models. The plan for the exhibition was to keep it open up to the end of August 2019. The gallery space was booked for another exhibition starting in the beginning of September but other premises were possible to use up to the end of October. All pieces that were implemented, were also strong enough to last during the whole period. Cultural producer Johanna Kallioaho and works manager Ari Virtanen with their teams were helpful and supported the set up that took place during one week.

The course lectures started at the beginning of January 2019. The opening event was agreed to take place at 21st May. To guarantee that the exhibits for the final exhibition were finished and ready by that date, a strict schedule had to be followed. This also meant that we had to adjust our lectures according to this schedule. Pablo Riquelme and Markus Holste were a great team to take the needed steps. Our earlier experience at Heureka Science Centre exhibition 2107 was taken

FIGURE 7. Venue of the exhibition

as a starting point with some modifications to the situation at hand. A calendar divided into four tasks and some subtasks was made for us to follow. All tasks were to be completed during sessions at the Design Factory, where each team gave their presentations and then the teachers, Sami Supply from EMMA and other students had the possibility to give their criticism. The presentations were also peer reviewed by other student groups and feedback was collected in written form. The critics events were important not only for the follow up of each team, but also to increase interaction between the groups and gain ideas for the atmosphere of the exhibition.

6. Team work

6.1. The first Task in mid January. The first task was divided in two parts. The first part was an individual work assignment to be finished independently. Regarding the course participant's own areas of interest or academic profile they were asked to observe and document for a period of time (10.01.2019-17.01.2019) different subjects that have an impact on their daily routine. The observations should be from a personal perspective, such as a visitor, artist, influencer, developer, etc. The participants were invited to collect at least 5 insights on how they experience architecture, art, design or mathematical expressions, every day for a period of 7 days. The purpose of this pre-group work assignment was to make the participants to pay attention to their surroundings and reveal insights leading to ideas and suggestions for further concepts. These pre-group work assignments materials were then used during the course process.

The students were asked to document different places, situations, phenomena that could have a relationship direct or indirectly with architecture, art, design

or mathematical expressions. Description of the impact of these observations on student's daily routine e.g. personal experience, practical effect, emotional impact, happiness, curiosity, etc. was expected. The media to be used for the documentation could be both written and/or visual material. The course participants were also encouraged to collect and bring supportive materials, such as photos, articles found on local news, etc. The outcome of this first part of the first task was to ask the participants to write a short analysis about the chosen topics, and present it to their own groups during the contact meeting in mid January. Especially, the students were expected to answer the following questions: Why did they choose these particular subjects? How does the subject affect the lives of the users (customer, client, other)? The participants were also asked to identify an intuitive/emotional response to address the experiences described.

The second part of the first assignment was done in groups by early February. Based on the individual assignment part, collective discussions and meetings, the student groups were asked to choose the most prominent subject / phenomena / interest area to be further developed into a concept that would serve as a base for the group creative work. The outcome of the first Task for each group was a 10 minutes long digital presentation exposing the following aspects. Why did the group choose this particular subject? How does the subject affect the lives of the users (customer, client, other)? How this subject is relevant for other people? Can the chosen subject be expressed or condensed on a single emotion? Can the chosen subject be scaled or multiplied? The presentations were expected to be both written and visually informative.

6.2. The second Task in late February. By the end of February the student groups were expected to find and present their concept as a statement of what they were going to be working on for the next months. The media or tools they were going to use later, was allowed to diverge but their concept would remain the same till the end of this creative process. The students groups were expected to produce max 10 minutes of visual and verbal presentation including short description of the chosen concept and analysis of the chosen approach underlining keywords that support the concept. Student groups were encouraged to visualize their ideas with mood boards, images, videos, sound etc. and be ready to defend the relevance of the chosen concept. Instructions for sharing the roles of the team mates, preparations for the presentations and their contents was also given. The second task was considered the most important one. This was the one that would give the direction of each artwork.

6.3. The third Task mid March. The third task was to bring each team's concept into a more realistic phase. The focus of this task was in SHAPE of the artworks. Student groups were asked to produce a scale model representing what their final piece was going to be. Students were allowed to use any material that suited to the concept and also use construction technics that were easy to handle within each team. It was not necessary to build the environment where the piece was going to be placed, but the students should go in detail about the following aspects: size, location for the installation, idea for the installation (on a pedestal, suspended from the ceiling, on a surface, etc.) and supportive infrastructure (audio, light, etc.). The technical details on how to install would then be developed together

with the available team. The models were instructed to be presented in scale 1:1 / 1:25 / 1:50 depending the size of the structures.

6.4. The fourth Task by the end of March. The final task was to decide about the MATERIALITY of the pieces. According to the form and aesthetics that students wanted to achieve, they were asked to research on suitable surface materials for their piece, The choices should then be presented among mockups and a further developed prototype. The prototypes were asked to be presented in scale 1:1 / 1:25 / 1:50 depending the size of the structures. Students were also reminded about the limited time we had before the exhibition took place. We encouraged students to strengthen the dynamics of their teams by dividing tasks, and building a strict work agenda fulfilling the assignment they will be given within their teams. The emphasis was on clear communication and trust, that would be the key elements during the last period of the course. The students would have lots of work to do and lots of things to learn and apply at the same time. This turned out to be the most challenging part for many groups, as the deadlines for their other projects was about the same time. Production issues, costs, quality, time-tables, storage and transportation challenges were all to be solved. On top of all this, the documentation challenges are always the last but not least to be solved.

7. Artworks

IN TRANSITION - Mathematics and Art student works are documented in the video by Kalle Kataila from Aalto communications [**13**]. Pablo Riquelme and Markus Holste from Aalto ARTS designed the exhibition set up at the Espoo Cultural Centre. The attached information below is edited from the public exhibition texts provided by the student groups. The texts were available on site next to the pieces and behind QR codes during the exhibition.

7.1. The Älvdans on the Moon Bridge. Student group Yi-Chiao Tien, Tomi Hyyppä and Alisa Kurganova from Aalto University School of Arts, Design and Architecture and Jannica Savander from Aalto University School of Electrical Engineering.

Revealing the void of the existing architecture by the Älvdans in the natural form of minimal surfaces.

The Älvdans on the moon bridge is a delightful piece which flows in between the void of the three-level staircase located in Tapiola Cultural Center, Espoo. Inspired by the weather phenomenon known as the "Älvdans" in Swedish, the students present it with fragmented minimal surfaces as the captured movement of the poetic scene, hoping to create a beautiful encounter for the audience and the existing space.

Materials: Power net, Metal components, Metal wire

Process: In the beginning stage of the project, the students were given a brainstorming task to observe and document inspirational phenomena in our surrounding. The students also had a one day workshop to explore and build up quick models on site- Tapiola Cultural Center. This group found interest in the void of the three-level staircase and the natural phenomena in the space such as air flow, heat transfer, and light. Since then they have been focusing on creating the image of a living creature flowing through space. "Älvdans", meaning "dance of

FIGURE 8. The Älvdans on the moon bridge. Photo by Kalle Kataila.

the elves", is a Swedish term describing the weather phenomenon that consists of local fog near the ground. Tapiola Cultural Centre designed by Arto Sipinen was named as Moon Bridge, which derived from the building's reflection in the Central Basin. Students were fascinated by the poetic link between the two, therefore, The Älvdans on the Moon Bridge was born.

As the students were searching for the form of the Älvdans, they found that minimal surfaces were ideal for their project. In mathematics, minimal surfaces have zero mean curvature and are originally minimized total surface area subject to some constraints. The constraints for the Älvdans in their project is the staircase area of the existing architecture. As a result, they can reveal the void of the existing architecture by the Älvdans in the natural form of minimal surfaces.

7.2. Flock in the Mountains. Student group: Lauri Neuvonen from Aalto School of Business, Jonna Tuupainen from Aalto University School of Arts, Design and Architecture, Nemanja Jovanovic from Aalto School of Electrical Engineering and Joel Saarinen from Aalto School of Science.

With help of Tapio Takala. Acknowledgements to Craig Reynolds for the Boids algorithm used as inspiration.

Materials: Translucent paper, Xbox Kinect Camera, Viewsonic Viewsonic LS830, Projector, black shade fabric, Raspberry Pi computer

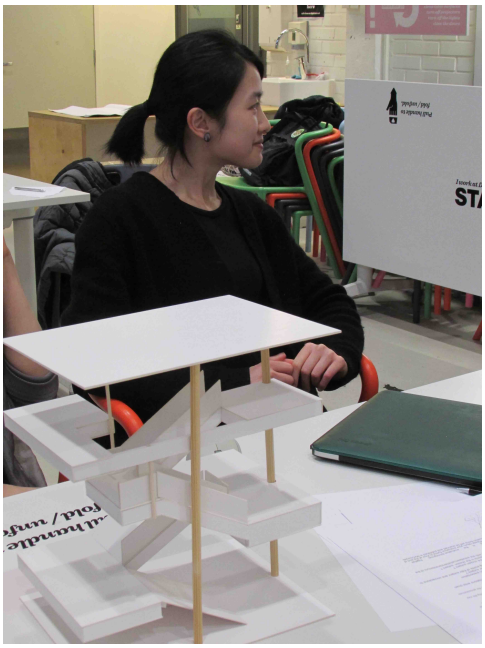


FIGURE 9. Model by Yi-Chiao Tien

Process: The students started off by thinking more generally about what phenomena in the world form emergent patterns. From automotive traffic to treacherous nonparallel waves in the ocean, emergence meets us frequently in our daily lives. Yet, by looking at emergence in biology, they discovered Conway's Game of Life, a computer game meant to generate emergent visual patterns by simulating the growth of cells. From this point on, an algorithm, such as the one used by Conway, seemed a promising way to explore emergent patterns through mathematics, since they usually consist of multiple components that could be adjusted and thus lead to an understanding of the phenomena through interaction. Using Conway's algorithm as an inspiration, the student group began to search for other emergent algorithms that allowed for some type of user interaction, and came across Craig Reynolds' "Boids" animation, which simulated the behaviour of flocking birds. After some tests, they found that giving the observers an opportunity to disrupt the flock of birds was a satisfying, yet simple way to feel the effects of emergence. By later adding a grid in the background of the animation, they hoped to better bring out the fact that each bird moves along a changing vector in the 2D space, showcasing the scene's mathematical underpinnings.

7.3. Ice Crystal. Students: Jennifer Greb and Vesa Putkonen from Aalto University School of Arts, Design and Architecture, Pinja Pessi and Päivi Putkonen from Aalto School of Science

With help of Eetu Enqvist, Paja&Bureau

Materials: The materials employed in this sculpture are steel and ice paper. The support structure for the crystal is welded from rectangular steel tubes. This robust

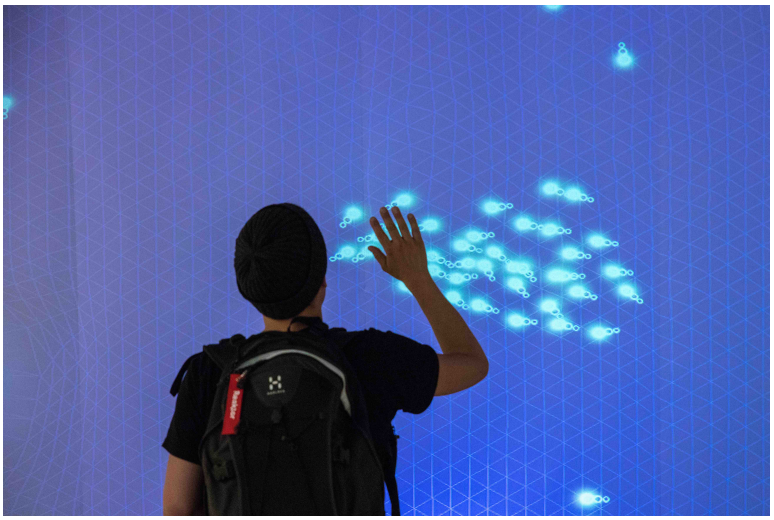


FIGURE 10. Flock in the Mountains. Photo: Mikko Raskinen.

material affords the piece its large size, spanning over four meters long. The surface of our sculpture was produced with ice paper. Ice paper is coated with adhesive containing salt that crystalizes. When the coating dries, it gives the material a frosted appearance. The transparency of the paper enables the light and shadows to enhance the form of the pattern.

Process: The sculpture magnifies mathematic principles which are found in ice crystals at a molecular level. The hexagonal shape of the sculpture in inspired by the natural form of a crystal, while the kirigami honeycomb fold emulates the pattern that the water molecules form when frozen.

To achieve the pattern on the surface of the sculpture the students utilized Kirigami. Kirigami is a variation of origami, the ancient Japanese art of paper folding. In addition to folding, kirigami requires the paper to be cut, allowing for interesting and complex 3-dimensional shapes. The honeycomb pattern was chosen for this sculpture because of its hexagonal shape, which mimics the molecular structure of ice.

The pattern for the kirigami-honeycomb pattern was first designed in a CAD program, and then converted to an Adobe Illustrator file. The file was utilized in a large blade clutter, which created mountain and valley folds, as well as cuts, on the ice paper. After the folds and cuts were produced, the paper was folded by hand to create the 3-dimensional pattern. The steel frame was welded to manufacture two interlocking pieces. Once the structure was assembled, students employed magnets to adhere the ice paper to the frame.

7.4. A fragment of infinity. Students: Heikki Humberg and Megan McGlynn from Aalto University School of Arts, Design and Architecture, Topi Nieminen from Aalto School of Science and Tuula Turunen from Aalto School of Business.

Assisting faculty members from Aalto or other helpers for this project where Hannu Paajanen, Mikko Ristimäki and Kenrick Bingham. Students also want to

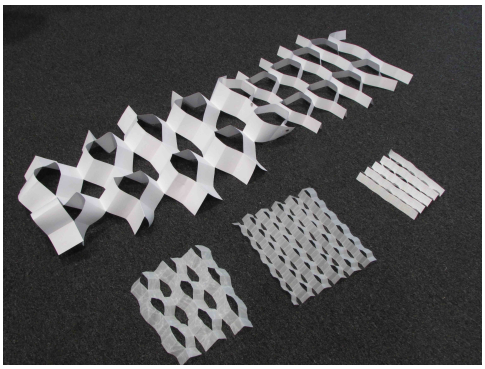


FIGURE 11. Ice Crystal mockups.

FIGURE 12. Ice Crystal. Photo: Kalle Kataila.

acknowledge the help of various schools in the area of Espoo and Vantaa for providing them with necessary extra Fresnel lenses.

Materials: Plywood, Fresnel lenses, Acrylic, Mirror, Dichroic Film, LEDs

The geometry of Fresnel lenses makes it a highly efficient method of refracting light. The sculpture distorts the classical Fresnel effect using a multi-faceted crystalline structure. The student group built a sculpture that plays with the eye using various different reflective and light manipulating materials. Fresnel lenses, dichroic films, mirroring surfaces and led lights together make the piece look different depending on the angle of the viewing and the time of the day.

FIGURE 13. A fragment of infinity. Photo: Kalle Kataila.

Process: The student group started with thinking about mirrors, what they represent in the western culture, the power of reflection and how the bending of the light is both a mathematically interesting and an aesthetically pleasing phenomenon. Soon enough they became fascinated about the possibilities to create an illusion using reflection as a tool. After benchmark research they decided to create a type of an infinity mirror. Due to Megan's previous work with polarizing filters they decided to also twist the concept of a traditional infinity mirror by adding filters to the mix. After a testing with various surfaces they decided to make an

infinity mirror piece, spiced up with dichroic filters and a Fresnel lens, for its beautiful effect on the light. The shape of the piece was a tough decision to make and took almost two weeks by itself. After arriving to the final shape students created the necessary 3D- and 2D-models and decided on the materials. They also acquired Fresnels with the help of the author and Kenrick Bingham, and they also got projectors with more lenses from various schools in Espoo and Vantaa. Because it was not possible to acquire one big Fresnel, out of which they would shape our piece, they created a Fresnel mosaic out of these smaller ones. They used a hexagonal pattern to build the mosaic because of its elegancy and space-filling abilities. For the frame of the piece they used plywood, because it is sturdy, does not change its shape, and is easy to mill. They designed the piece so that the 3D- acrylic structure would be supported by the frame and little stress would be directed to the seams of the acrylic. The sealed the seams with acrylic glue, which would not be too visible. It was important for the surface to project the full Fresnel effect.

7.5. Growing pains. Student: Linda Mandell Aalto University School of Arts, Design and Architecture. With help of Felix Bade and Lasse Naukkarinen. Materials: Ceramics, aluminium

The sculptures depict growth as a natural phenomenon as well as mathematical concept through transforming geometry. The clusters of polyhedrons are arranged in a grid based on Archimedean tiling, forming bodies that imitate the growth of a slime mold. Mold's ability to find breaches where it can claim its space and thrive showcases the disruptive force that growth has, and through the process of making it also grew to represent my own personal development.

Process: The first ideas about the work was discussed with a team member. They discussed about the relationship of mathematics and art and how to bring humane aspect to mathematical art. They were fascinated by the idea of disruptive interference and highlighting details that are easily left unseen. They ended up talking about mold and its relentless ability find breaches where it can claim its space and thrive, and how it showcases the force that growth has. Through these discussions they realised that growth can be interpreted in many ways, especially as a natural phenomenon as well as mathematical concept.

FIGURE 14. Linda's ceramics mockups

Even though Linda ended up executing the final sculptures herself, these discussions were the starting point to the final work. The design process started through discussions and sketching, iterating, fast sculpting and more sketching. At a quite early point she knew she wanted to make the final piece from ceramics.

After the final presentations the design had turned from crystal-like growth to polyhedrons arranged in a grid based on Archimedean tiling, forming bodies that imitate the growth of a slime mold. Fast prototyping with paper gave clear visuals of what the shapes should be, as well as some ideas about what can be done with them. Paul Jackson's folding workshop acted as a great practice to produce these types of shapes with ease. When the pattern, the shapes and the scale was decided, the ceramic process began. Slip casting requires many steps, beginning with models. For this design, Linda created five models, three of them of the sculpted from plaster and two were 3D-printed. The next step was to make molds from plaster and to save time in the slip casting part she decided to make nine molds altogether. This enabled her to really try serial production and made the casting easy. After casting the pieces, the bottoms have to be finished by grinding them so that they fit together.

Ceramics are often bisque fired in lower temperatures before the glazing so that the pieces will not break so easily, but Linda decided to save time and glaze the pieces without bisque firing them first. The glazing required some experiments with different pigments and finishes, and she settled 10% sun yellow pigment with semi-matte glazing. The burning temperature for the glazing was 1240°C, and it usually takes around 24 hours for the oven to reach that temperature and cool down afterwards.

When there were enough pieces ready, Linda started to think about the shapes of the clusters. She wanted to find the right organic rhythm and texture for each cluster, and still keep the structure open for possible changes. To be able to do this required as much casting of the pieces as possible in the time that was available, and she ended up casting over 300 individual pieces. The water jet cut, aluminium back pieces of each cluster were designed based on the groups for the display. One cluster had strict measurements as it was going to be place in a certain place, so there was a different approach to the design. Overall this part of the process was very intuitive.

Just before the gluing and the last decisions about the design, Linda booked the studio, where she could have some fun with the pieces and have final feel how they should be handled. This was also a way to document these pieces before they stick to only one position, and to give them freedom to organise themselves.

7.6. The Crystal Garden. Students: Punit Hiremath and Iiro Törmä from Aalto School of Arts, Design and Architecture and Saara Vestola from Aalto School of Science.

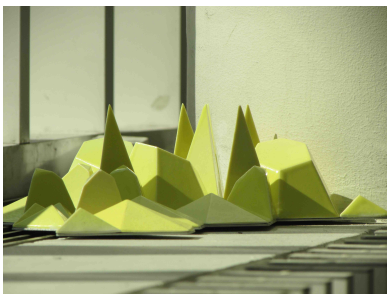
With help of Janne Ojala.

Materials: Acrylic and metal sheets, led lights, mirrors

Flowers are an abstraction of the real flowers and are represented by polygons. The work of art represents the life cycle of the flower. "Crystal Garden" is linked to Tapiola as the Garden City.

Process: Nature and its phenomena were in the minds of this student group from the very beginning of their project. Why does snow glitter? Why is there life

(A) Growing Pain assembly.



(B) Growing Pain on the window.

(C) Growing Pain on the wall. Photo: Kalle Kataila.

FIGURE 15. Growing Pain.

cycles in the nature? How light is the source of life? How it behaves and how people observe its behaviour? They wanted to bring light, time, reflections and shadows to their installation and they ended up with an idea of an installation that would represent the interface of nature and light. In the beginning the biggest problem was: How could they make an abstraction of nature and make it interesting? The inspiration for the installation was firstly sparked by the luminous "Moonwatch tower" of the Espoo Cultural Centre. Especially the students were attracted to the third floor space and from that day they have known where their installation would be located. In that space there must be something big. All they made after

that visit was focused on that special space. The name of the installation, Crystal Garden, is connected to Tapiola as the Garden City.

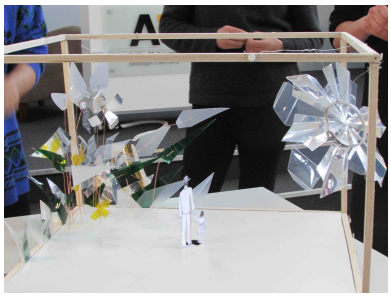
Students made many drawings and were inspired by abstract flowers and polygons in those. In very short time they had first shape of their flower. Mathematics is present in their work through polygons and symmetries. The shape of all the petals is same, only size and folding differs. Through the form of a geometrical and transparent flower that is multiplied, scaled and varied in the space the students tried to create a simple abstraction of nature.

There are four big flowers and many little ones in the installation. Those four big flowers represent the circle of life. First flower with red inner petals is just born closed bud. Second one, a young flower, with orange inner petals has already opened a little and the third one, the biggest with yellow inner petals, is the grown up flower. Fourth flower has already seen life and has lost its colour. Petals are also petals of the old flower. Main material, acrylic, was chosen because of its transparency and reflectivity. Acrylic sheets were also easy to laser cut and fold with heat to the shape we wanted. Mirrors in the center of the flowers are there to add reflections.

8. Student feedback on Math&Arts Minor

Arts student Megan McGlynn was one of the first students who participated Aalto Mathematics and Arts Minor, that was piloted during 2018-2019. Below her talk at the opening of the IN TRANSITION exhibition at Espoo Cultural Centre in May 2019.

My background is in fine art and architecture - and though those things require some spatial reasoning and geometry - the last time I actually studied math was in high school. In my experience, math has a bad reputation with "artsy" people or "creative types" (broad generalizations). Generally, math is avoided at all costs. It is a scary world that has way too many rules- rules that you must be able to immediately grasp or you're "just not good at it". You hear this a lot "I'm just good at math" - but that's not how it works. It's like a language, anyone's who's willing to learn can start to understand it. Of course, it might take some time to learn the basics, but you don't need to be fluent to start understanding some things, making connections, and possibly opening up new ways of thinking. I think that's exactly what these courses are about - they don't require rigorous mathematical study or really any previous knowledge. We were given an overview of many topics, almost all of which had some spatial or visual component beyond numbers and variables. It gave us the opportunity to look further into whatever topic we found most interesting. The topics included things like: projective geometry, the 4th spatial dimension, the math and analysis of pattern-making, fractals, topology and categorization of shapes, folding (or origami), the list goes on and on.... These concepts gave us a sense of how math can be an inspiration for art and design, even in a light hearted way. That you don't need to be an expert to explore these things - you can ask simple questions, play with materials, analyse existing knowledge and possibly find interesting solutions. We learned a few examples of artists have actually made significant contributions to mathematics, and mathematicians have made some great artists. And that is another very positive thing about these courses - that they're interdisciplinary. It has always been my belief that some of the best art is created, not



(A) A mock-up of Crystal Garden.

(B) Figure 30 Iiro Törmä and some flowers. Photo: Kalle Kataila

(C) Crystal Garden. Photo: Kalle Kataila.

FIGURE 16. Crystal Garden.

from studying art-making itself, but from people creatively exploring their own fields of study. Even better is when people can work together across different fields, to push and pull,, add and merge knowledge and skills to create something truly transformative. I think this is was Aalto is trying to achieve with bringing the Arts and Business schools to the campus in Otaniemi. Courses like the math/arts courses are a great step in fostering these important connections between schools. Lastly, Not only do these courses give art and design students more confidence to work with concepts of mathematics, (and hopefully for mathematics students to work with designers) - but through the work we make, hopefully it gives that confidence to much wider audiences - to show everyone that math isn't this insulated, incomprehensible world. But it's everywhere, it's elegant, and it's exciting. So we hope that everyone is enjoying the work we've made for this exhibition, and that maybe you'll be inspired to explore further on your own.

(A) Student Tuula Turunen and her dance group at the opening event. Photo: Mikko Raskinen.

(B) Megan McGlynn at the opening of the exhibition Photo: Mikko Raskinen.

9. IN TRANSITION goes to Shanghai November 2019

Aalto Math&Arts in Shanghai Future Art Lab exhibition [**3**] was a joint effort of Aalto School of Arts, Design and Architecture, Aalto School of Science and Aalto School of Engineering. The focus of the contribution of Aalto University was to introduce the interdisciplinary Math&Arts program, especially its underlying course Crystal Flowers in Halls of Mirrors: Mathematics, Arts and Architecture and related activities to the audience. The Future Lab exhibition of Aalto, in figure 18 , curated and produced by Laura Isoniemi, consisted of three sections showcasing the development of Aalto Math&Arts program, the Past, Present and Future concept of our activities at Aalto University. The Past section told the story of the Aalto Math&Arts program through videos, posters, graphical info material, hands on educational models and ideas related to surface design. The Present part continued the story by showing the latest course outcomes from the exhibition IN TRANSITION – Mathematics and Art at Espoo Cultural Centre in Finland via textiles from workshops of Isoniemi and an interactive virtual exhibition realized by Juho-Pekka Virtanen from Memo Research group [**24**] at Aalto School of Engineering. The virtual exhibition combines conventional online content with panoramic images, illustrations, augmented reality and 3D scenes. By utilizing the techniques from 3D geomatics, such as photogrammetry and laser scanning, the virtual exhibition allows the user to study the IN TRANSITION –exhibition contents and also creates a lasting digital footprint for the otherwise temporary event. The virtual exhibition can be accessed online at link [**14**].

The Future part communicated the pedagogical ideas implemented with students of Tongji-Huangpu School of Design & Innovation, teachers from different schools and universities in China and showcased a large-scale model designed by Taneli Luotoniemi. This sculpture, SPACE HUG, invited the visitor to join a non-stop installation process and build his/her own small scale model from bamboo sticks.

Several workshops and lectures were organized during our Math&Arts visit to Shanghai. Tiina Laurila, Head of Creative Curriculum Development, Associate Professor and Senior Adviser for the Tongji College of Design and Innovation organized all these interaction possibilities for us. Two lectures about curvature was arranged for first grade high school students at Tongji-Huangpu School of Design & Innovation.

FIGURE 18. Aalto Math&Arts booth at Shanghai FutureLab. Photo: Juho-Pekka Virtanen

FIGURE 19. Visitors exploring the virtual exhibition on a touch screen. Photo by Juho-Pekka Virtanen.

Two whole day sessions were organized for college student from different backgrounds and levels. Lectures were give on Thurston-Conway classification of planar and spherical symmetries. This theoretical part was completed by hands on folding activities and textile block printing by Laura Isoniemi. The final outcomes of this workshop were curated as a part of our FutureLab exhibition. Taneli Luotoniemi gave also lectures about 4D and ideas about its visualizations.

A session for teachers of Tongji-Huangpu School of Design & Innovation and other schools and universities in China was organized to present general ideas around Aalto Math&Arts. As an example, we went through some samples of educational origami and hands on activities with bamboo sticks. Teachers were also invited to show their outcomes during the exhibition next week and also show random visitors how to build the structures they learned.

Aalto Math&Arts visit to Shanghai was funded by Dean Tuomas Auvinen from School of Arts, Design and Architecture, Dean Jouko Lampinen from School of Science and Dean Gary Marquis from School of Engineering. The collaborators in Shanghai were West Bund Art Center, Sino-Finnish Centre, students and teachers from Tongji-Huangpu School of Design & Innovation and teachers from other schools in Shanghai, Beijing and other provinces of China as well as students from Tongji University.

We hope to be able to describe our Shanghai experience in more detail elsewhere in the near future.

Acknowledgement

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